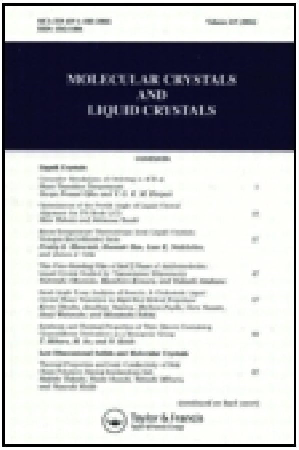

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Defect States in a Nematic Liquid Crystal in a Magnetic Field

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The different defect configurations that are possible in nematic liquid crystals in a magnetic field have been presented explicitly. They have been compared with defects that occur in magnetic spin systems.

Keywords: defects in magnetic fields, disclinations, solitons, point defects

1. INTRODUCTION

A nematic liquid crystal with diamagnetically positive anisotropic molecules has two permissible states of orientation in an external magnetic field. The director can be either along or opposite to the field direction in a medium of infinite extent. These two permitted states are connected through a wall whose properties were studied first in detail by Helfrich.[1] These walls are very similar to the Bloch or the Néel walls of spin systems. We can have a bend-rich or a splay-rich wall or even a pure twist wall. On crossing the wall the director orientation changes by $\pm\pi$. Volovik and Mineev studied such domains walls and showed[2,3] that they can end in disclination lines of half integral strength. They have been termed as planar solitons. It was also argued that a cylindrical domain wall can end in a hedgehog point singularity. This has been designated as a linear soliton.[2,3] Kléman's work[4–6] on the structure and properties of Néel and Bloch disclinations in small anisotropy ferromagnetic systems and the article on domain walls by Malozemoff and Slonczewski[7] are also of relevance to the present study. These studies point to the possible existence of many more defect states in nematics, which appear not to have been explicitly discussed in the above references. In this paper we present these situations which bring in a closer analogy with magnetic systems. It should be remarked that these new defect states are probably of higher energy compared to walls.

2. LINE DISCLINATIONS

λ-Disclinations

In the one constant approximation all the three different types of walls have the same energy as well as thickness. This permits us to think of a wall of one kind to transform itself into another. In Figure 1 a bend Néel wall has been transformed into a Bloch wall. The two walls are separated by a λ disclination. This is a familiar process in magnetic systems.[5] It must be pointed out that not only can one have positive or negative λ disclinations, but one can also think of a further subdivision depending upon whether the Bloch wall is right- or left-handed.

η-Disclinations

In Figure 2 we show a splay-Néel wall being joined to a Bloch wall through another type of disclination which may be called an η-Disclination. This line is topologically equivalent to a ±1/2 wedge line.

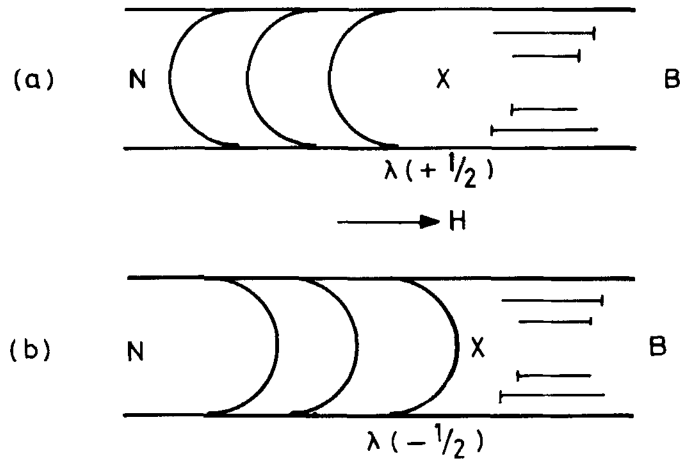

FIGURE 1 Bend-Néel wall to Bloch wall transformation through a λ (±1/2) disclination. The cross indicates the line defect.

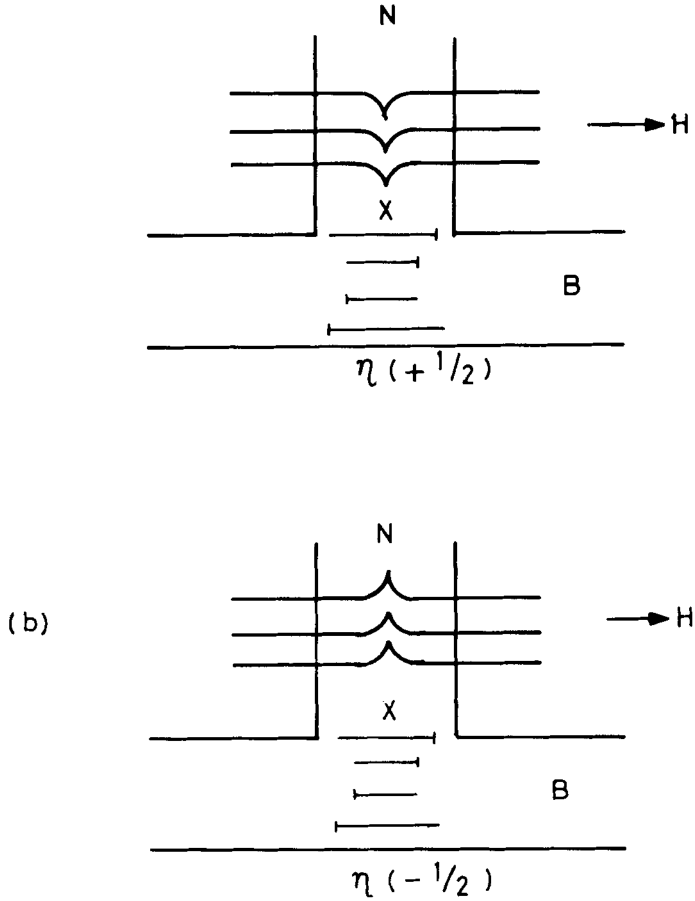

FIGURE 2 Splay-Néel wall joining a Bloch wall through an η-disclination.

δ-Disclinations

They will be similar to η excepting for the Bloch wall being replaced by a bend Néel wall.

Néel and Bloch lines

Line defects can also be expected in a Bloch or a Néel wall. They separate regions of exactly opposite twist, bend or splay deformations. Again such disclinations have been suggested in magnetic systems. These can be of wedge or twist type.

We notice that in a Bloch wall the director twists through $\pm\pi$ as we cross it from one side to another. Since both $+\pi$ and $-\pi$ walls are equally probable we have to consider their simultaneous existence as well. Such oppositely twisted regions get connected through a twist disclination line of unit strength. Such lines are designated as Néel lines as in magnetism. We have shown in Figure 3 two possible configurations for the Néel line. In Figure 3(a) it is perpendicular to the field, and in Figure 3(b) it is parallel to it. In general it can even be at an angle, but detailed energetics may lock it into one of the two directions only.

In Figure 4 we have depicted a similar situation for a bend Néel wall. In classical magnetic systems configurations Figures 4(a) and (b) have been postulated. They have respectively $s = +1$ and $s = -1$ wedge disclination lines separating regions of opposite distortion. They have been termed Bloch lines in magnetism. In Figure 4(c) is shown one other possibility of the Bloch line being parallel to the field. In this case the Bloch line is of twist type.

In Figure 5 similar situations are given for a splay Néel wall. Here again we can have wedge or twist Bloch lines.

3. BUBBLE DOMAINS

Symmetry of nematic liquid crystals permits one to consider cylindrical shell structures separating the inside and outside regions through

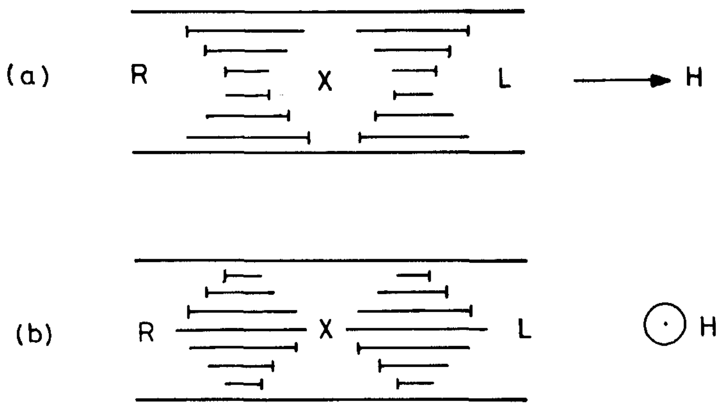

FIGURE 3 Néel lines in a Bloch wall. In (a) it is perpendicular to the field H. In (b) it is parallel to it. Field perpendicular to the plane of the figure is shown by a circle with a dot.

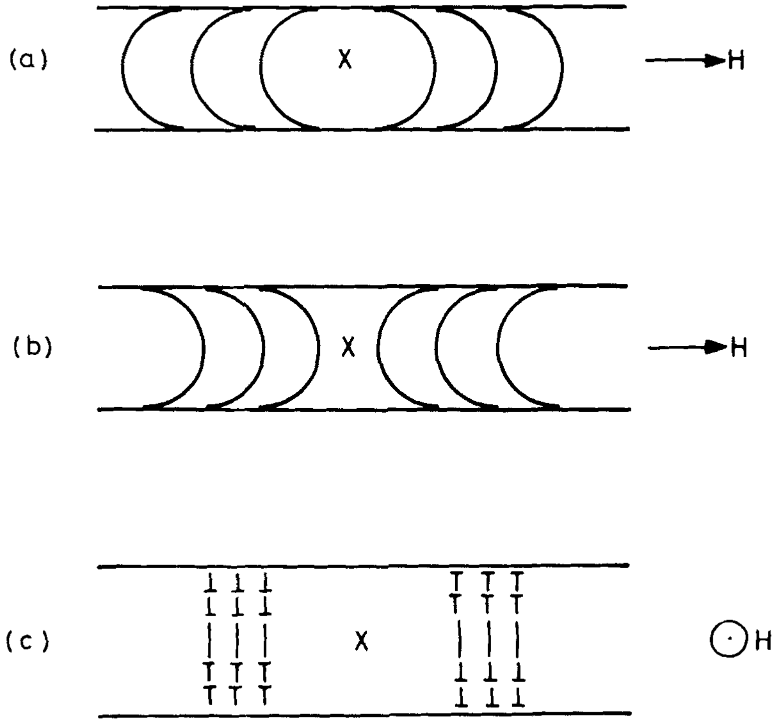

FIGURE 4 Bloch line in a bend-Néel wall. (a) +1 line and in (b) −1 line. In both these cases the line is perpendicular to H. (c) Bloch line is parallel to H and is a twist disclination line.

twist or bend distortions. These are equivalent to rolling Bloch or Néel walls into cylinders [Figures 6(a) and (b)]. As we cross the cylinder from inside to outside or vice versa the director again turns through π. They can be called bubbles. And just as Bloch or Néel walls can end in disclinations, we can think of these bubbles ending in disclination rings [Figures 7(a) and (b)].

It is easy to see that when a Néel bubble ending in a ring shrinks, we end up with a linear soliton[2,3] first discussed by Mineev and Volovik. Whether a bubble decays into a linear soliton or vice versa is to be settled only through detailed energy calculations. Similar arguments hold for a Bloch bubble ending in a ring.

We can also think of ring defects in bubbles. For example a Bloch bubble can go over to a Néel bubble through a λ-ring disclination [Figure 7(c)]. When such a structure is allowed to collapse we get a Néel type linear soliton going over to a Bloch type linear soliton,

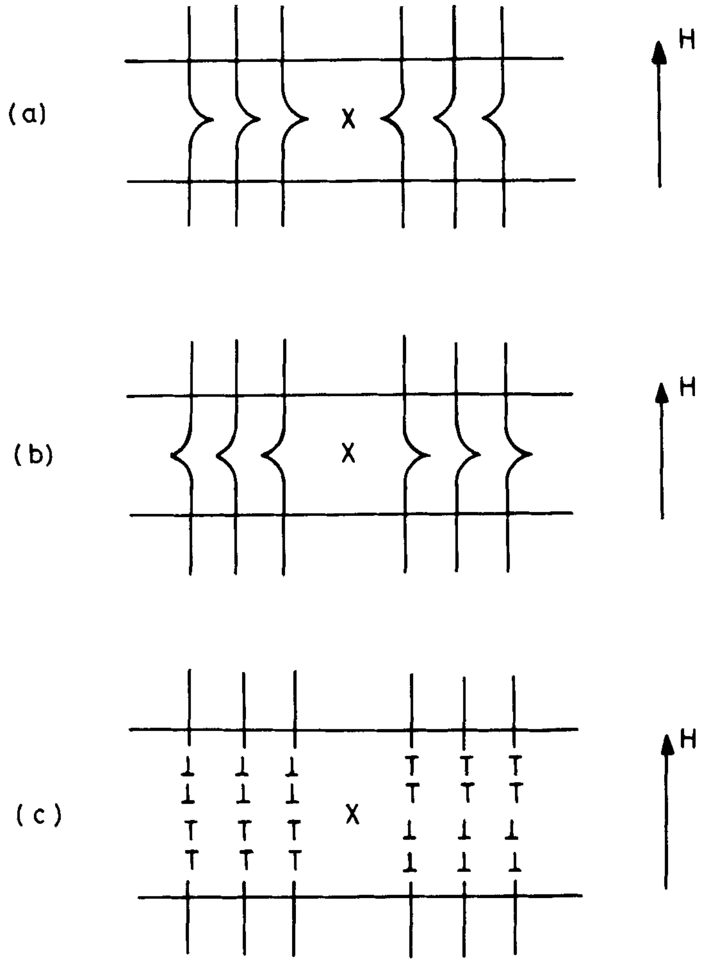

FIGURE 5 Bloch lines in splay-Néel walls. (a) +1 line, (b) −1 line, (c) twist line. In all the three cases the line is perpendicular to H.

through a point singularity. Likewise we can connect bubbles of exactly opposite twist or bend deformations through Néel or Bloch ring disclinations.

Line defects can be constructed even in bubbles. We show in Figure 8(a) a Bloch bubble having an unlike pair of Néel lines separating regions of opposite twists. In Figure 8(b) we give a similar possibility for a Néel bubble.

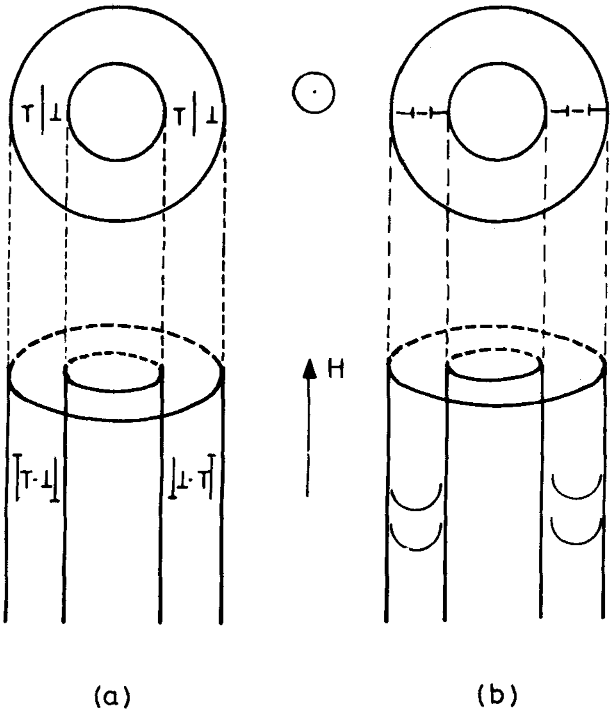

FIGURE 6 (a) Bloch bubble, (b) Néel bubble.

4. POINT SINGULARITIES

We know that an hedgehog can go over to a linear soliton in a magnetic field.[2,3] This transformation is mostly through the bend deformation. However, we find a very different structure if we look for solutions that are rich in splay. If we insist on splay to dominate the distortions in the magnetic field then we find cases shown in Figure 9. [We can arrive at the same pattern by looking for cylindrically symmetric splay Néel walls.] In Figure 9(a) we have shown a hedgehog and in Figure 9(b) a hyperbolic point singularity.

Here P will be a singular point in what can be called a disc. On crossing the disc the director turns through $\pm\pi$ and thus it is much like a wall. Thus a point singularity can transform itself into a disc under magnetic field instead of becoming a linear soliton. As to which actually is the solution is decided by the energetics of the deformations contained in the two situations.

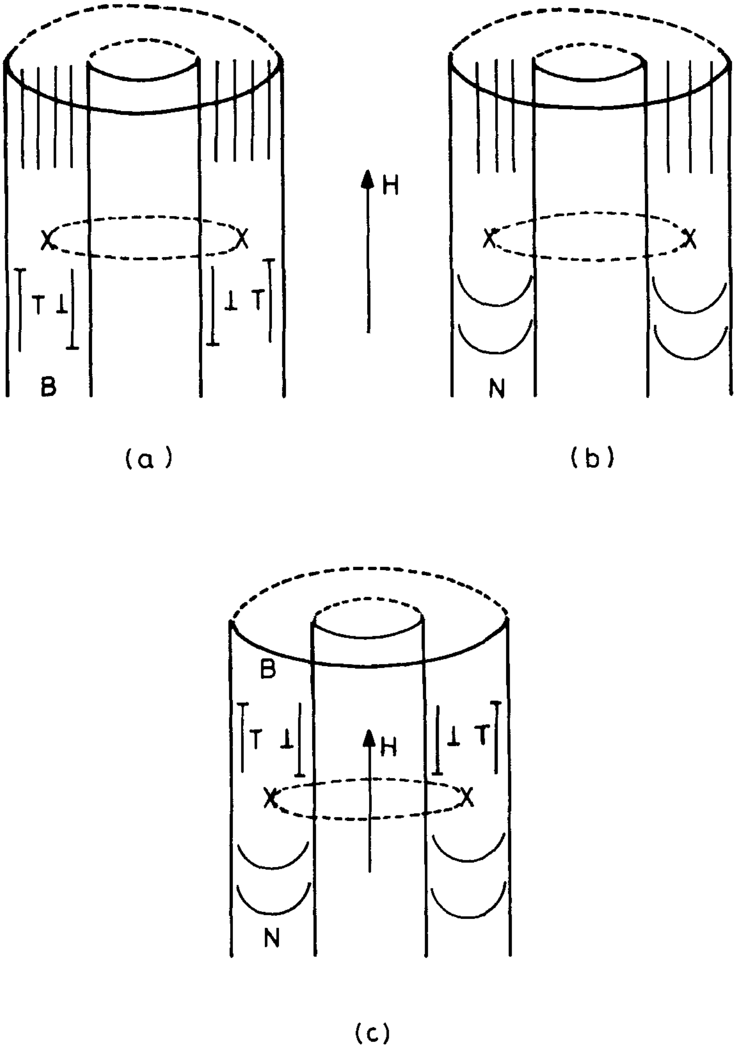

FIGURE 7 (a) Bloch bubble ending in a twist ring disclination, (b) Néel bubble ending in a wedge ring disclination, (c) a Néel bubble transformation to a Bloch bubble through a λ ring disclination.

Nematics can also have half integral Poincaré point defects of the type discussed by Nabarro[8] and Blaha.[9] These are really not isolated point defects but end points of disclination lines of strength ±1. In fact a +1 hedgehog can split into two Poincaré +1/2 defects connected by +1 disclination line. Also we can eliminate a +1 discli-

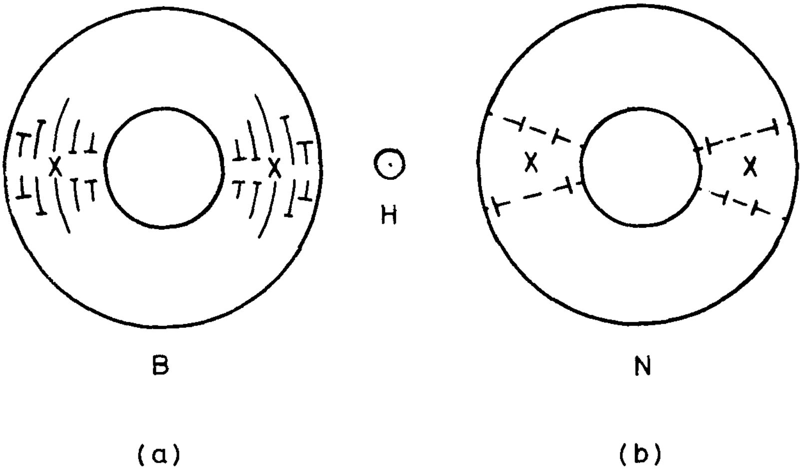

FIGURE 8 (a) Néel lines in a Bloch bubble, (b) Bloch lines in a Néel bubble.

nation line through a thread discontinuity with the creation of an unlike Poincaré defect pair. This is analogous to an exactly similar possibility with a hedgehog and a hyperbolic point at the thread discontinuity.[10]

In Figures 10(a) and (b) are shown transformations of Poincaré $\pm 1/2$ point defects in a magnetic field. In both the cases the whole

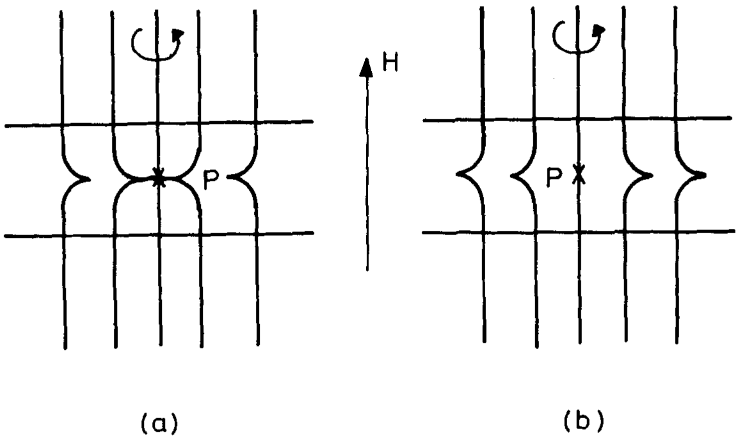

FIGURE 9 (a) Transformation of an hedgehog into a disc. This is mostly through splay deformation. (b) Similar transformations in an hyperbolic point.

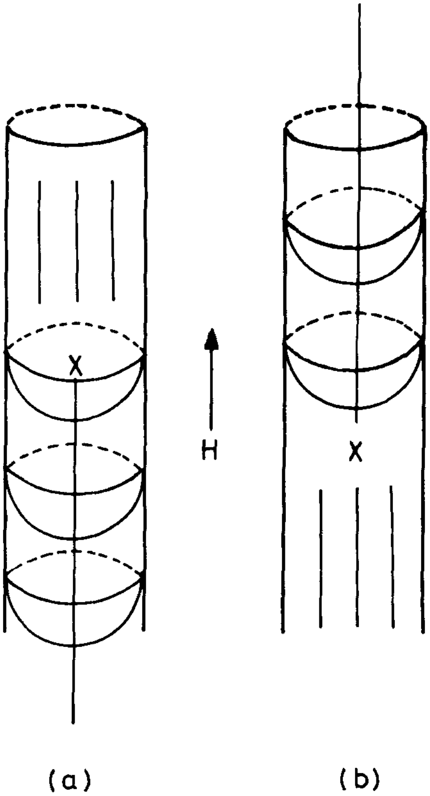

FIGURE 10 Half integral Poincaré defects in a magnetic field. (a) $+1/2$ wedge defect, (b) $-1/2$ wedge defect.

of deformation is confined to a cylinder in space and the cylinder ends in a point defect. To this extent they are similar to linear solitons. But, in addition, we also have singular lines of strength $+1$ through the center of the tube. Also in crossing the cylinder radially the director orientation changes by $\pm\pi$ in contrast to familiar linear solitons where this change is through $\pm 2\pi$. We can call them Poincaré solitons.

Acknowledgments

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